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HELSINGIN MATEMATIIKKALUKIO

Name _____ Group _____

Points: _____ Kangaroo leap: _____

Separate this answer sheet from the test.

Write your answer under each problem number.

For each wrong answer, 1/4 of the points of the problem will be deducted.

If you don't want to answer a question, leave the space empty and no deduction will be made.

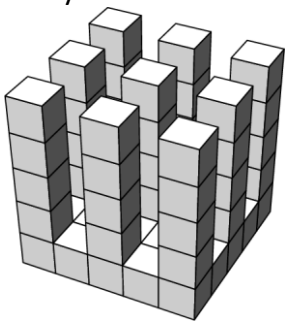
PROBLEM	1	2	3	4	5	6	7	8	9	10
ANSWER										

PROBLEM	11	12	13	14	15	16	17	18	19	20
ANSWER										

PROBLEM	21	22	23	24	25	26	27	28	29	30
ANSWER										

3 points

1.
If you take a number of $1 \times 1 \times 1$ cubes out of a $5 \times 5 \times 5$ cube, you end up with a solid figure consisting of columns of the same height, which stand on the same ground plate (see figure). How many small cubes were taken out?



(A) 56 (B) 60 (C) 64 (D) 68 (E) 80

2.
Paula's digital watch is slightly broken. The rightmost digit of the display has lost all three of its horizontal lines. Paula looks at her watch just as the display changes from the figure on the left to the figure on the right. What is the time now?

(A) 12:40 (B) 12:42 (C) 12:44 (D) 12:47 (E) 12:49

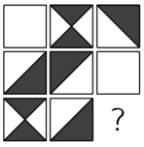
3.
A cake weighs 900 g. Panu cuts it into four pieces. The largest piece is as heavy as the three smaller ones combined. How much does the largest piece weigh?

(A) 250 g (B) 300 g (C) 400 g (D) 450 g (E) 600 g

4.
Today is the birthday of Carla, Emilie and Lilian. The sum of their ages is 44 years today. Some years from now there will come a birthday when the sum on their ages will have two same digits again. What is the sum of their ages then?

(A) 66 (B) 77 (C) 88 (D) 99 (E) 100

5.
Which tile must be added to make the black and white areas equal?



(A)  (B)  (C)  (D)  (E) It is impossible

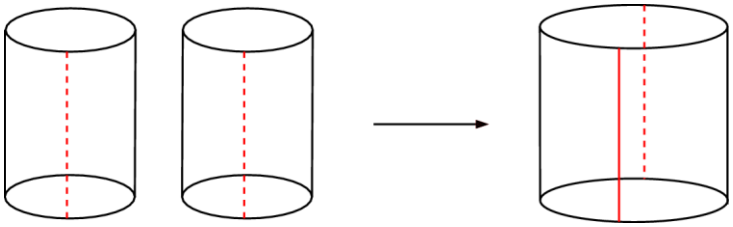
6.
Which of the following expressions does not have $b + 1$ as a factor?

(A) $2b + 2$ (B) $b^2 - 1$ (C) $b^2 + b$ (D) $-1 - b$ (E) $b^2 + 1$

7.
There are 24 balls placed into three baskets of different sizes. The smallest and the largest basket contain together twice the number of balls that the middle one contains. The smallest basket contains half the number of balls of the middle one. How many balls are there in the largest basket?

(A) 8 (B) 10 (C) 12 (D) 15 (E) 16

8.
Two identical cylinders are cut open along the dotted lines and glued together to form one bigger cylinder -- see figure. What can you say about the volume of the big cylinder compared to the volume of one small cylinder?



(A) It has twice the volume.
(B) It has 3 times the volume.
(C) It has π times the volume.
(D) It has 4 times the volume.
(E) It has 8 times the volume.

9.
How many digits are there in the result of the calculation $(2^{22})^5 \cdot (5^{55})^2$?

(A) 22 (B) 55 (C) 77 (D) 110 (E) 111

10.
In the number 2014 the digits are different and the last digit is greater than the sum of the other three digits. How many years ago did this occur the last time?

(A) 5 (B) 215 (C) 305 (D) 395 (E) 485

4 points

11.
Handsome Harry has a secret email account that only four friends know. Today he received 8 emails in that account. Which of the following is certainly true?

(A) Harry received two emails from each friend.
(B) Harry cannot have received eight emails from one of his friends.
(C) Harry received at least one email from each friend.
(D) Harry received at least two emails from one of his friends.
(E) Harry received at least two emails from 2 different friends.

12.
It is known that $a^b = \frac{1}{2}$. What is a^{-3b} ?

(A) $\frac{1}{8}$ (B) 8 (C) -8 (D) 6 (E) $\frac{1}{6}$

13.
The size of a rectangular cardboard box is $a \times b \times c$. It is known that $a < b < c$. If you increase a or b or c by a given positive number, the volume of the box also increases. In which of the following cases is the increase of the volume of the box the greatest?

(A) If you increase a.
(B) If you increase b.
(C) If you increase c.
(D) The increase of the volume is the same in (A) – (C)
(E) It depends on the values of a, b, c.

14.

$$\frac{2^{2014} - 2^{2013}}{2^{2013} - 2^{2012}} = ?$$

(A) 2^{2011} (B) 2^{2012} (C) 2^{2013} (D) 1 (E) 2

15.
The king and his messengers are travelling from the castle to the summer palace at the speed of 5 km/h. Each hour a messenger travelling 10 km/h leaves the group and rides back for the castle. What is the time interval between any two consecutive messengers arriving at the castle?

(A) 30 min (B) 60 min (C) 75 min (D) 90 min (E) 120 min

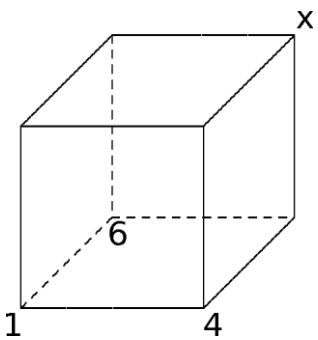
16.
Six weeks is $n!$ seconds. What is n?
(The notation $n!$ stands for the product $1 \cdot 2 \cdot 3 \cdot \ldots \cdot n$.)

(A) 6 (B) 7 (C) 8 (D) 10 (E) 12

17.
How many triplets of integers (a, b, c) satisfy $a > b > c > 1$ and $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} > 1$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) infinitely many

18.
The vertices of a cube are numbered 1 to 8 in such a way that the result of adding the four numbers of the vertices of a face is the same for all faces. Numbers 1, 4 and 6 are already set on some vertices as shown. What is the value of x?

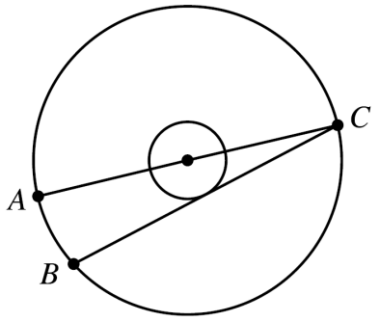


(A) 2 (B) 3 (C) 5 (D) 7 (E) 8

19.
Function $f(x) = ax + b$ satisfies $f\left(f\left(f(1)\right)\right) = 29$ and $f\left(f\left(f(0)\right)\right) = 2$. What is a?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

20.
The radii of two concentric circles are in proportion 1 : 3.



AC is a diameter of the big circle; BC is a chord of the big circle and a tangent to the smaller; and the length of AB is 12. Then the radius of the big circle is

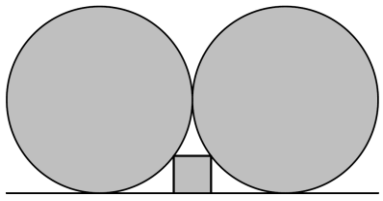
(A) 13 (B) 18 (C) 21 (D) 24 (E) 26

5 points

21.
There are 10 different positive integers on a blackboard, exactly 5 of which are divisible by 5 and exactly 7 of which are divisible by 7. Let M be the greatest of these 10 numbers. What is the minimum possible value of M?

(A) 105 (B) 77 (C) 75 (D) 63 (E) something else

22.
A square fits snugly between two touching circles of radius 1 and the horizontal line which is tangent to both of the circles. What is the side length of the square?



(A) $\frac{2}{5}$ (B) $\frac{1}{4}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\frac{1}{5}$ (E) $\frac{1}{2}$

23.
Tom wants to write several distinct positive integers, none of them exceeding 100. Their product should not be divisible by 54. At most how many integers can he write?

(A) 8 (B) 17 (C) 68 (D) 69 (E) 90

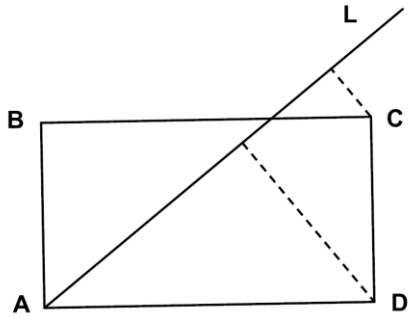
24.
Two regular polygons lie on opposite sides of their common side AB. One of them is a 15-gon $ABCD$ … and the other is an n-gon $ABZY$ …. What value of n makes *CZ* = *AB* ?

(A) 10 (B) 12 (C) 15 (D) 16 (E) 18

25.
The label on a package of cream cheese reads: 24 % total fat. The same label also reads: 64 % fat in dry matter. What is the percentage of water in this cheese?

(A) 88 % (B) 62,5 % (C) 49 % (D) 42 % (E) 37,5 %

26.
Line L passes through the vertex A of a rectangle $ABCD$. The distance from point C to L is 2, and the distance from point D to L is 6. Also $AD = 2AB$. How long is AD?

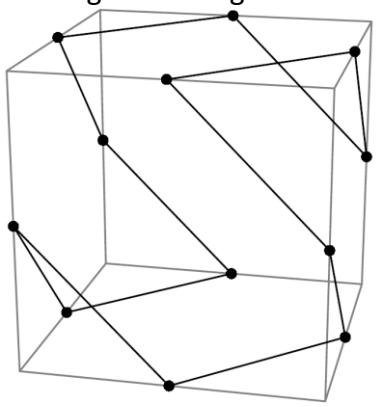


(A) 10 (B) 12 (C) 14 (D) 16 (E) $4\sqrt{3}$

27.
There are 9 kangaroos called Greatkangs. They are coloured either silver or golden. When 3 Greatkangs meet by chance, there is a 2/3 chance that none of them is silver. How many Greatkangs are golden?

(A) 1 (B) 3 (C) 5 (D) 6 (E) 8

28.
The diagram shows a 3-dimensional polygon whose vertices are the mid-points of the edges of a cube. An angle of the 3-dimensional polygon is defined in the normal way: the angle between the two edges meeting at a vertex. What is the sum of all the interior angles of the polygon?



(A) $720°$ (B) $1080°$ (C) $1200°$ (D) $1440°$ (E) $1800°$

29.
The function $f: \mathbb{Z} \to \mathbb{Z}$ satisfies the conditions
$f(4) = 6$ and
$xf(x) = (x-3)f(x+1)$.

What is the value of $f(4) \cdot f(7) \cdot f(10) \cdot \ldots \cdot f(2011) \cdot f(2014)$?

(A) 2013 (B) 2014 (C) $2013 \cdot 2014$ (D) 2013! (E) 2014!

30.
In the forests of a magical island three kinds of animals roam: lions, wolves and goats. Wolves can eat goats, and lions can eat either wolves or goats. However, this being a magical island:

1. If a wolf eats a goat, it turns into a lion.
2. If a lion eats a goat, it turns into a wolf.
3. If a lion eats a wolf, it turns into a goat.

Originally, there were 17 goats, 55 wolves and 6 lions on the island. What is the highest possible number of animals remaining on the island after no more eating is possible to happen?

(A) 1 (B) 6 (C) 17 (D) 23 (E) 35