NAME ________________________ GROUP ______

Points: _____ Kangaroo leap: _____

Separate this answer sheet from the test.

Write your answer under each problem number.

For each wrong answer, 1/4 of the points of the problem will be deducted.

If you don't want to answer a question, leave the space empty and no deduction will be made.

PROBLEM	1	2	3	4	5	6	7	8	9	10
ANSWER										

PROBLEM	11	12	13	14	15	16	17	18	19	20
ANSWER										

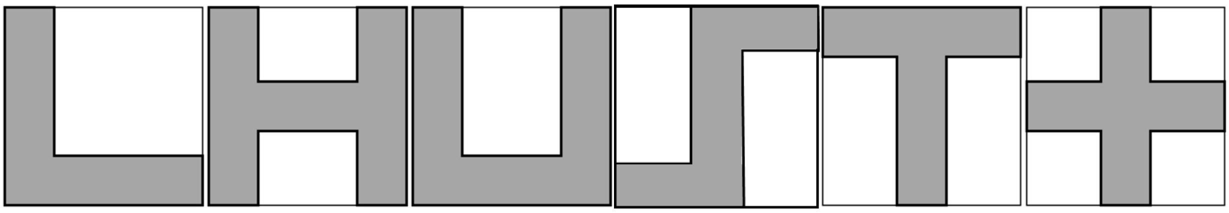
PROBLEM	21	22	23	24	25	26	27	28	29	30
ANSWER										

3 points

1.
Which of the following numbers is the largest?

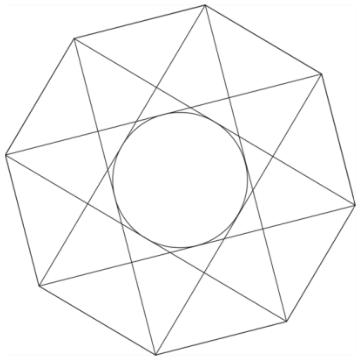
(A) 2013 (B) 2^{0+13} (C) 20^{13} (D) 201^3 (E) $20 \cdot 13$

2.
Mary drew figures on identical, square paper sheets. How many of these grey figures have perimeter equal to the perimeter of the paper sheet?



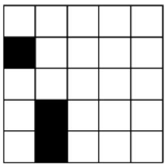
(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

3.
In the figure is a regular octagon with side lenght 10. How long is the radius of the circle inscribed in the smaller octagon formed by the diagonals?



(A) 10 (B) 7,5 (C) 5 (D) 2,5 (E) 2

4.
Carina and a friend are playing a game of "battleships" on a 5×5 board. Carina has already placed two ships as shown. She still has to place a 3×1 ship so that it covers exactly three cells. No two ships may touch, not even at corners. How many positions are there for her 3×1 ship?



(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

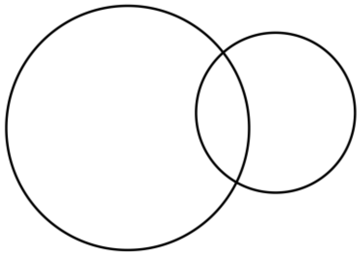
5.
Six superheroes capture 20 villains. The first superhero captures one villain, the second captures two villains and the third captures three villains. The fourth superhero captures more villains than any of the other five. What is the smallest number of villains the fourth superhero could have captured?

(A) 7 (B) 6 (C) 5 (D) 4 (E) 3

6.
The year 2013 has the property that its number is made up of the consecutive digits 0, 1, 2 and 3. How many years have passed since the last time a year was made up of four consecutive digits?

(A) 467 (B) 527 (C) 581 (D) 693 (E) 990

7.
By drawing two circles, Mike obtained a figure, which consists of three regions (see picture).



If he had drawn squares instead, how many regions could he have gotten at most?

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

8.
A prism has 2013 faces in total. How many edges does the prism have?

(A) 2011 (B) 2013 (C) 4022 (D) 4024 (E) 6033

9.
The cube root of 3^{3^3} is equal to

(A) 3 (B) 3^{3^3-1} (C) 3^{2^3} (D) 3^{3^2} (E) $(\sqrt{3})^3$

10.
Given that $2 < x < 3$, how many of the following statements are true?

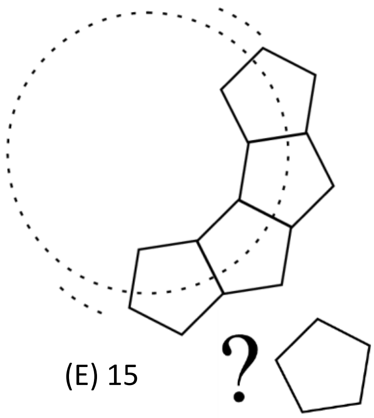
$$4 < x^2 < 9 \qquad 4 < 2x < 9 \qquad 6 < 3x < 9 \qquad 0 < x^2 - 2x < 3$$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

4 points

11.
Radu has identical plastic pieces in the shape of a regular pentagon. He glues them edge to edge to complete a circle - as shown in the picture.

How many pieces are there in this circle?



(A)8 (B) 9 (C) 10 (D) 12 (E) 15

12.

When a certain solid substance melts, its volume increases by $\frac{1}{12}$. By how much does its volume decrease when it solidifies again?

(A) $\frac{1}{10}$ (B) $\frac{1}{11}$ (C) $\frac{1}{12}$ (D) $\frac{1}{13}$ (E) $\frac{1}{14}$

13.

How many positive integers n exist such that both $\frac{n}{3}$ and $3n$ are three digit integers?

(A) 12 (B) 32 (C) 34 (D) 100 (E) 300

14.

The function f is defined for all real x by the following two properties:

1. f is periodic with period 5
2. on the interwall $[-2, 3[$ it holds that $f(x) = x^2$.

What is $f(2013)$?

(A) 0 (B) 1 (C) 2 (D) 4 (E) 7

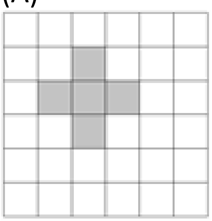
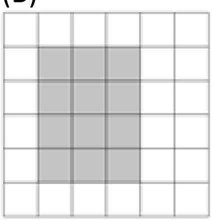
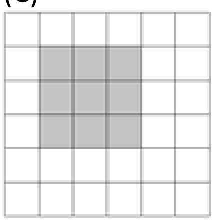
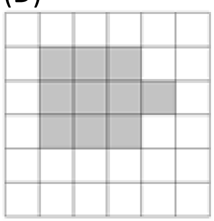
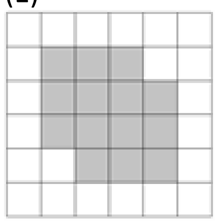
15.

Einari and Pauliina are arguing about a function f on the set of integers.
Einari claims: "For any even n, $f(n)$ is even."
It turned out that Einari was wrong. What must be true?

(A) For any even n, $f(n)$ is odd
(B) For any odd n, $f(n)$ is even
(C) For any odd n, $f(n)$ is odd
(D) There exists an even number n such that $f(n)$ is odd
(E) There exists an odd number n such that $f(n)$ is odd

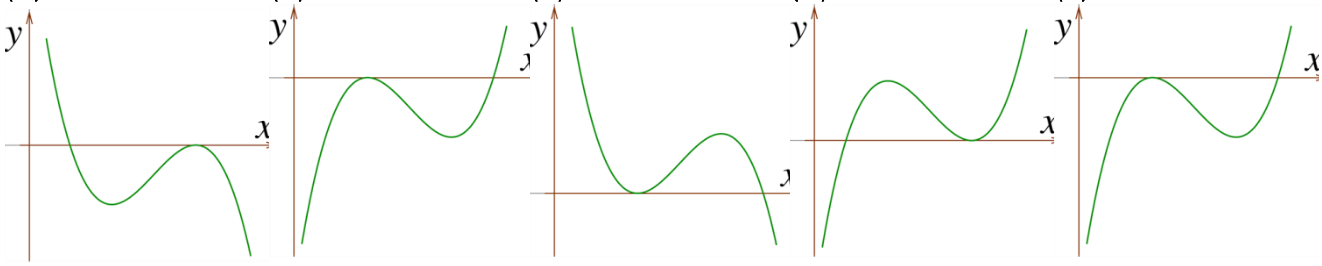
16.

A circular carpet is placed on a floor of square tiles. All the tiles which have more than one point in common with the carpet are marked grey. Which of the following is an impossible outcome?

(A)

(B)

(C)

(D)

(E)


17.
We are given a function $W(x) = (a - x)(b - x)^2$, where $a < b$. Its graph is in one of the following figures. In which one?

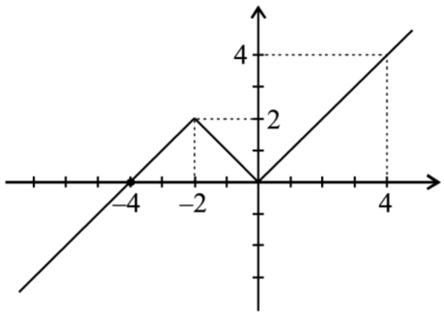
(A) (B) (C) (D) (E)



18.
Consider a rectangle, one of whose sides has length 5. The rectangle can be cut into a square and a rectangle, one of which has an area of 4. How many such rectangles exist?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

19.
Vlad has drawn the graph of a function $f: R \longrightarrow R$, composed of two rays and a line segment (see figure). How many solutions does the equation $f(f(f(x))) = 0$ have?



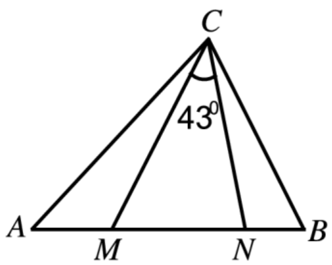
(A) 4 (B) 3 (C) 2 (D) 1 (E) 0

20.
A box contains 900 cards numbered from 100 to 999. Any two cards have different numbers. François picks some cards and determines the sum of the digits on each of them. At least how many cards must he pick in order to be certain to have three cards with the same sum?

(A) 51 (B) 52 (C) 53 (D) 54 (E) 55

5 points

21.
In the triangle ABC the points M and N on the side AB are such that $AN = AC$ and $BM = BC$. Find $\angle ACB$ if $\angle MCN = 43°$.



(A) 86° (B) 89° (C) 90° (D) 92° (E) 94°

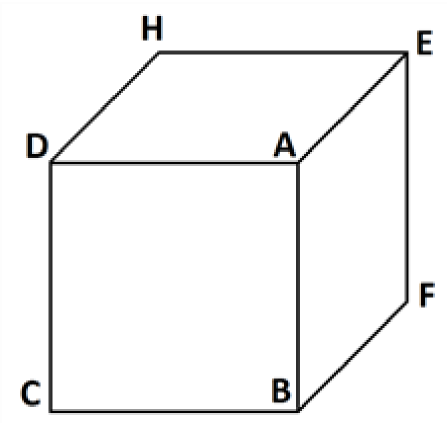
22.

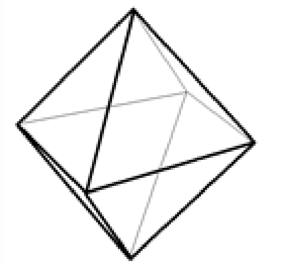
Let f be a function on the set of integers defined by $f(n) = \frac{n}{2}$ if n is even, $f(n) = \frac{n-1}{2}$ if n is odd, for all integers n. For k a positive integer $f^k(n)$ denotes the number represented by the expression $f(f(\ldots f(n)\ldots)))$, where the symbol f appears k times. The number of solutions of the equation $f^{2013}(n) = 1$ is

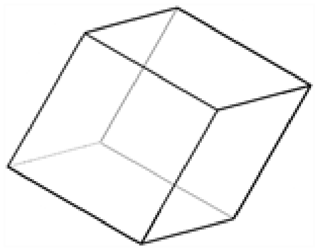
(A) 0 (B) 4026 (C) 2^{2012} (D) 2^{2013} (E) infinite

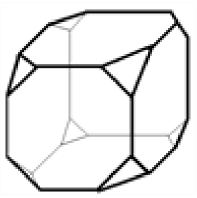
23.

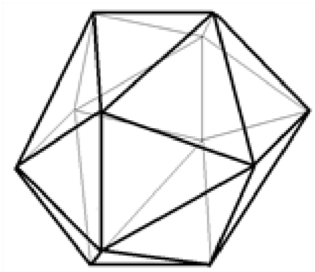
The solid cube in the figure is cut by a plane passing through the three neighbouring vertices (D, E and B) of vertex A. Similarly the cube is cut by planes passing through the three neighbouring vertices of all other seven corners. After all the cutting the pieces are separeted. What will the piece containing the center of the cube look like?



(A) 

(B) 

(C) 

(D) 

(E) The center of the cube belongs to several pieces.

24.

The sum of the first n positive integers is a three-digit number in which all of the digits are the same. What is the sum of the digits of n?

(A) 6 (B) 9 (C) 12 (D) 15 (E) 18

25.

How many solutions (x, y), where x and y are real numbers, does the equation $x^2 + y^2 = |x| + |y|$ have?

(A) 1 (B) 5 (C) 8 (D) 9 (E) Infinitely many

26.
There are some straight lines drawn on the plane. Line a intersects exactly three other lines and line b intersects exactly four other lines. Line c intersects exactly n other lines, with $3 \neq n \neq 4$. Determine the number of lines drawn on the plane.

(A) 4 (B) 5 (C) 6 (D) 7 (E) Another number

27.
How many pairs (x, y) of integers with $x \leq y$ exist such that their product equals 5 times their sum?

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

28.
Iulian has written an algorithm in order to create a sequence of numbers as

$$a_1 = 1, \qquad a_{m+n} = a_m + a_n + mn,$$

where m and n are natural numbers. Find the value of a_{100}.

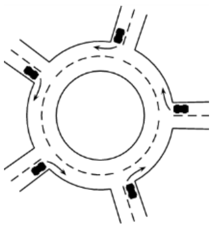
(A) 100 (B) 1000 (C) 2012 (D) 4950 (E) 5050

29.
On the island of Knights and Knaves there live only two types of people: Knights (who always speak the truth) and Knaves (who always lie). I met two men who lived there, and asked the taller man if they were both Knights. He replied, but I could not figure out what they were. Next I asked the shorter man if the taller was a Knight. He replied, and after that I knew exactly which type they were.

Were the men Knights or Knaves?

(A) They were both Knights.
(B) They were both Knaves.
(C) The taller was a Knight and the shorter was a Knave.
(D) The taller was a Knave and the shorter was a Knight.
(E) Not enough information is given.

30.
The roundabout shown in the picture is entered by 5 cars at the same time, each one from a different direction. Each of the cars drives less than one round and no two cars leave the roundabout in the same direction. How many different combinations are there for the cars leaving the roundabout?



(A) 24 (B) 44 (C) 60 (D) 81 (E) 120