Kangaroo 2010 Junior
(gymn. 1st year)

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HELSINGIN MATEMATIIKKALUKIO

NAME ________________________ CLASS/GROUP ______

Points: _____ Kangaroo leap: _____

Separate the answer sheet from the test.
Write your answer under each problem number.
If you do NOT know the answer, leave the space empty.
For each WRONG answer, 1/4 of the points is deducted. DO NOT guess!

PROBLEM	1	2	3	4	5	6	7	8	9	10
ANSWER										

PROBLEM	11	12	13	14	15	16	17	18	19	20
ANSWER										

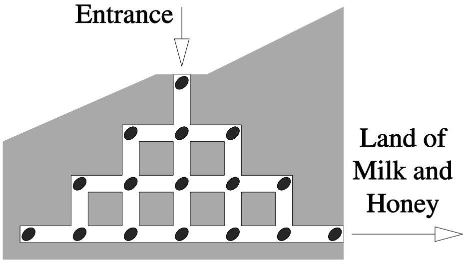
PROBLEM	21	22	23	24	25	26	27	28	29	30
ANSWER										

3 points

1.
My calculator divides instead of multiplying and substracts instead of adding. I type (12x3)+(4x2). What does the calculator show?

(A) 2 (B) 6 (C) 12 (D) 28 (E) 38

2.
Hamster Fridolin sets out for the Land of Milk and Honey. His way to the legendary Land passes through a system of tunnels. There are 16 pumpkin seeds spread through the tunnels, as shown in the picture.



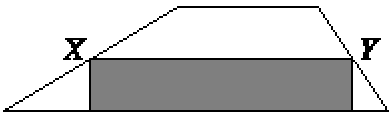
What is the highest number of pumpkin seeds Fridolin can collect if he is not allowed to visit any junction more than once?

(A) 12 (B) 13 (C) 14 (D) 15 (E) 16

3.
A zebra crossing has alternate white and black stripes, each of width 50 cm. The crossing starts and ends with a white stripe and has 8 white stripes in all. What is the total width of the crossing?

(A) 7 m (B) 7.5 m (C) 8 m (D) 8.5 m (E) 9 m

4.
The area of the shaded rectangle is 13 cm^2. X and Y are the midpoints of the sides of the trapezium.



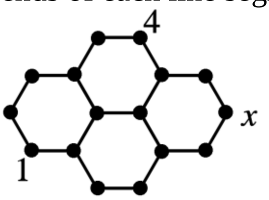
What is the area of the trapezium?

(A) 24 cm^2 (B) 25 cm^2 (C) 26 cm^2 (D) 27 cm^2 (E) 28 cm^2

5.
Given that $P = 2\cdot 3+3\cdot 4+4\cdot 5$, $Q = 2^2+3^2+4^2$ and $R = 1\cdot 2+2\cdot 3+3\cdot 4$, which of the following statements is true?

(A) $Q < P < R$ (B) $P < Q = R$ (C) $P < Q < R$ (D) $R < Q < P$ (E) $Q = P < R$

6.
A number is written at each of the dots of the lattice shown so that the sum of the numbers at the ends of each line segment is the same.



Two of the numbers have already been written. What number goes in the place labelled x?

(A) 1 (B) 3 (C) 4 (D) 5 (E) more information is needed

7.
When 2011 was divided by a certain number, the remainder was 1011. Which of the numbers 100, 500 and 1000 was the divisor?

(A) 100 (B) 500 (C) 1000
(D) some other number (E) it is not possible to get this remainder

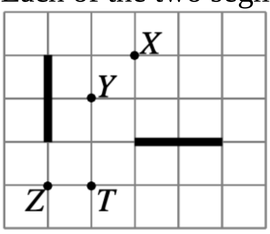
8.
A rectangular mosaic with area 360 cm^2 is made from square tiles, all the same size. The mosaic is 24 cm high and 5 tiles wide. What is the area of each tile?

(A) 1 cm^2 (B) 4 cm^2 (C) 9 cm^2 (D) 16 cm^2 (E) 25 cm^2

9.
Every 4-digit number whose digits add up to 4 is listed in descending order. In which place in the list is the number 2011?

(A) 6th (B) 7th (C) 8th (D) 9th (E) 10th

10.
Each of the two segments shown is a rotation of the other one.

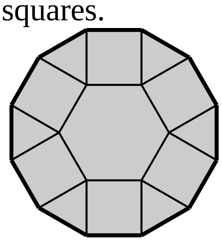


Which of the marked points may be the centre of such a rotation?

(A) Only X (B) X and Z (C) X and T (D) Only T (E) X, Y, Z and T

4 points

11.
The figure shows a shape consisting of a regular hexagon of side one unit, six triangles and six squares.



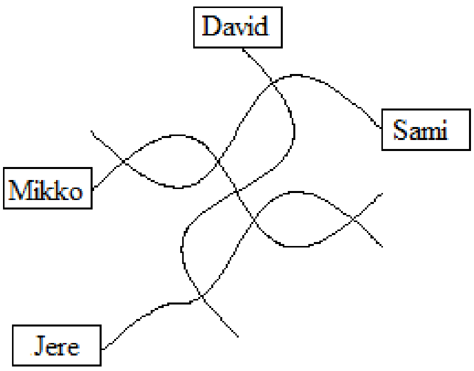
What is the perimeter of the shape?

(A) $6(1+\sqrt{2})$ (B) $6\left(1+\frac{\sqrt{3}}{2}\right)$ (C) 12 (D) $6+3\sqrt{2}$ (E) 9

12.
In a normal dice the sum of opposite faces is seven. Three normal dice are glued on top of each other so that the total number of pips on two faces glued together is always 5. One of the visible faces on the bottom die has one pip. How many pips are on the top face on the top die?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

13.
During a rough sailing trip, Joni tried to sketch a map of his home village, but had problems doing that because of the waves. He managed to draw four streets, their seven crossings and the houses of his friends.



In reality Arrow Street, Nail Street and Ruler Street are all perfectly straight. The fourth street is Curvy Road. Who lives on Curvy Road?

(A) David (B) Jere (C) Mikko (D) Sami (E) a better map is needed to be able to tell

14.
Three sportsmen participated in a race: Michael, Fernando and Sebastian. Immediately after the start, Michael was in the lead, Fernando was second, and Sebastian third. During the race, Michael and Fernando changed places 9 times, Fernando and Sebastian did so 10 times, and Michael and Sebastian did so 11 times. In what order did they finish?

(A) Michael, Fernando, Sebastian (B) Fernando, Sebastian, Michael (C) Sebastian, Michael, Fernando (D) Sebastian, Fernando, Michael (E) Fernando, Michael, Sebastian

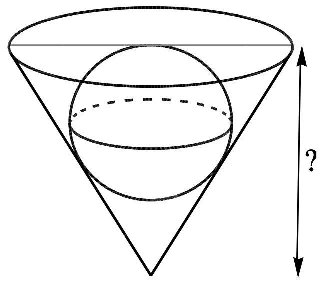
15.
Given that $9^n + 9^n + 9^n = 3^{2011}$, what is the value of *n*?

(A) 1005 (B) 1006 (C) 2010 (D) 2011 (E) none of these

16.
In a certain month there were five Mondays, five Tuesdays and five Wednesdays. In the preceding month there were only four Sundays. Which of the following will the next month definitely include?

(A) exactly 4 Fridays (B) exactly 4 Saturdays (C) 5 Sundays (D) 5 Wednesdays (E) this situation is impossible

17.
A sphere with radius 15 is rolled into a conical hole, which it just fits, as shown.



Viewed from the side the hole is an equilateral triangle. How deep is the hole?

(A) $30\sqrt{2}$ (B) $25\sqrt{3}$ (C) 45 (D) 60 (E) $60(\sqrt{3}-1)$

18.
Each cell of the 4×4-grid shown is to be coloured black or red.

				2
				0
				1
				1
2	0	1	1	

The number next to each row and column indicates how many cells in that row or column have to be black. In how many ways can this be done?

(A) 0 (B) 1 (C) 3 (D) 5 (E) 9

19.
How many numbers are there in the longest run of consecutive 3-digit numbers, each of which has at least one odd digit?

(A) 1 (B) 10 (C) 110 (D) 111 (E) 221

20.
Jaakko wants to write an integer in each cell of the 3×3 grid shown, so that the sum of the numbers in every 2×2 square is 10.

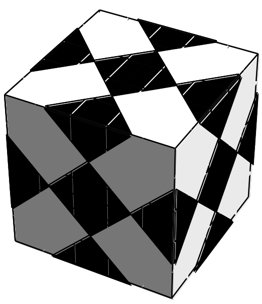
1		0
	2	
4		3

Five numbers have already been written. What is the sum of the four missing numbers?

(A) 9 (B) 10 (C) 11 (D) 12 (E) 13

5 points

21.
Sameli had a white plastic cube with edges of length 1 dm. He glued several congruent black squares on the cube, as shown, so that the cube looked the same on every face.



What was the total black area?

(A) 37.5 cm^2 (B) 150 cm^2 (C) 225 cm^2 (D) 300 cm^2 (E) 375 cm^2

22.
A five-digit number is defined ”suitable” if its digits are distinct and the first digit is equal to the sum of the other four digits. How many suitable numbers are there?

(A) 72 (B) 144 (C) 168 (D) 216 (E) 288

23.
The numbers x and y are both greater than 1. Which of the following fractions has the greatest value?

(A) $\frac{x}{y+1}$ (B) $\frac{x}{y-1}$ (C) $\frac{2x}{2y+1}$ (D) $\frac{2x}{2y-1}$ (E) $\frac{3x}{3y+1}$

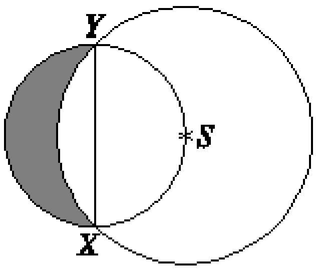
24.
How many different ordered pairs of natural numbers (x, y) satisfy the equation $\frac{1}{x}+\frac{1}{y}=\frac{1}{3}$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

25.
For an integer $n \geq 2$ denote by $\langle n \rangle$ the biggest prime number (an indivisible number greater than one) which does not exceed n. How many positive integers k satisfy the equation
$\langle k+1 \rangle + \langle k+2 \rangle = \langle 2k+3 \rangle$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) more than 3

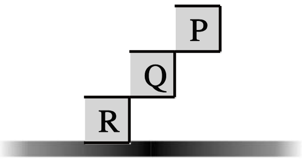
26.
Two circles are constructed as shown in the figure.



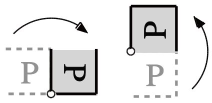
The segment XY is a diameter of the smaller circle and the centre S of the larger circle lies on the smaller circle. The radius of the larger circle is r. What is the area of the shaded region?

(A) $\frac{\pi}{6}r^2$ (B) $\frac{\sqrt{3}\pi}{12}r^2$ (C) $\frac{1}{2}r^2$ (D) $\frac{\sqrt{3}}{4}r^2$ (E) some other answer

27.
Anita is playing a computer game involving squares, starting from the position shown.



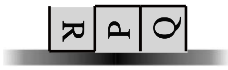
At each move, one square is turned through 90 degrees about a corner, as shown in the two examples.



The aim is to arrange the squares somewhere along the bottom of the screen.
Which of the following arrangements can Anita achieve?

(A) 

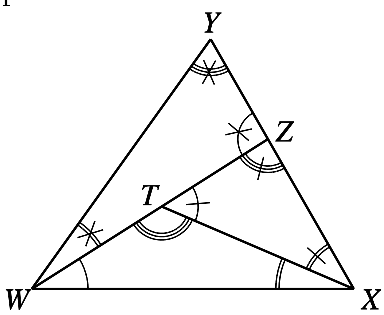
(B) 

(C) 

(D) 

(E) all of A-D are possible

28.
In triangle *WXY*, a point *Z* is chosen on the segment *XY*, then point *T* is chosen on the segment *WZ*, as shown. The figure is drawn so that the marked angles have as few different values as possible.



What is the smallest possible number of different values that the nine marked angles could take?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

29.
How many sets of four edges of a cube have the property that no two edges in the set have a common vertex?

(A) 6 (B) 8 (C) 9 (D) 12 (E) 18

30.
For which values of *n*, where $0 < n \leq 9$, it is possible to mark some cells in a 5×5 square in such a way that every 3×3 square contains exactly *n* marked cells?

(A) 9 (B) 1 and 9 (C) 1, 2, 3 and 9 (D) 1, 2, 8 and 9 (E) all values from 1 to 9